longitudinal forms realized in the shell in the zone adjacent to folds formed from the direction of the immovable support.

The good agreement can be seen in Fig. 2 of calculations and experiments for relationship N = N(t) up to quite large values of shell compression (≈40%) with which the given filler still does not have a marked effect on longitudinal force realized in the shell. The calculated nature of shape change for a cylindrical shell at different instants of time (after 100 μsec) for test No. 4 is given in Fig. 3. By comparing Fig. 3 with Fig. 2f over time it is possible to note that the increase in N = N(t) up to a critical value is observed with deflections exceeding the shell thickness, and a drop is observed with intense fold formation.

Comparison of calculated results with experimental data shows quite good agreement both for residual shell shape (see Figs. 1b, c and Fig. 3), and for the relationship N = N(t) (see Fig. 2), which points to the efficiency of the model suggested in describing shock compression of cylindrical shells of moderate thickness (h/R = 1/10...1/5).

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PLASTIC MODELS IN PROBLEMS OF ELASTIC DEFORMATION OF ROLLED SHELLS

S. V. Lavrikov and A. F. Revuzhenko

UDC 539.3

1. The questions considered in this work arose from the following considerations. We refer to classical solution of the Lamé problem for a thick-walled cylindrical tube. In view of axial symmetry for the problem tangential stresses are absent: $\sigma_{r\theta} = 0$ (r and θ are polar coordinates). This means that if an arbitrary number of cuts is made in the tube over the circumference r = const, then these cuts do not impinge on the operation of the structure. Consequently, the cross section of the tube may be represented by a collection of thin individual rings mounted close to each other; rings operate so that conditions at contacts between them do not affect the operation of the whole structure. As is well known, in this scheme the material is loaded very unevenly, and if the external radius of the tube exceeds the internal radius by more than a factor of three to four then a further increase in tube thickness has practically no effect on the change over of the inner region into a plastic condition (failure). Therefore, an idea occurs naturally: is it possible to organize the work of elastic rings in such a way that external friction forces are mobilized between them which would contribute to "resisting" external pressure. We cut up rings over a certain radius and glue them together with displacement by one pitch (Fig. 1). The structure obtained differs in principle from the previous one. It might be expected that as a result of slippage of layers it will be possible to include in the operation material distant from the inner boundary, and consequently to distribute the applied load more uniformly thus incresing the supporting capacity of the structure.

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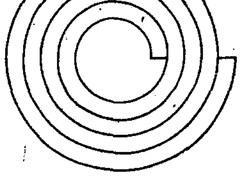

Fig. 1

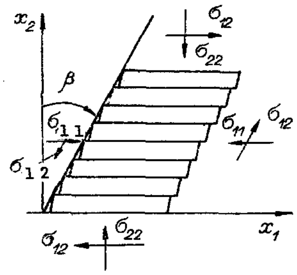

Fig. 2

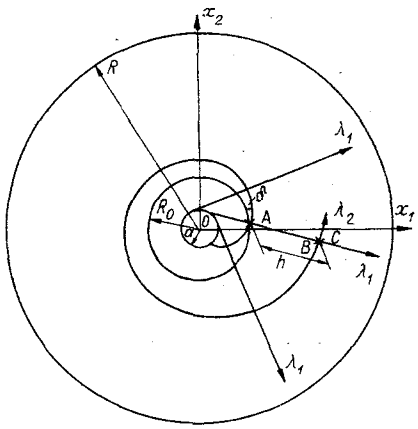

Fig. 3.

2. Let there be an elastic shell of constant thickness coiled into a roll (Fig. 1). It is assumed that the condition is natural. We take an individual element of the material including a certain number of layers of the roll. We do not yet consider the curvature of the contact line (Fig. 2). How does this element react to external loadings? It is evident that with compression in directions Ox_1 and Ox_2 in no way do discontinuities develop and element reaction will be elastic. Now let tangential stresses be applied, they give rise to elastic shear of material layers between discontinuities and in the general case to a certain amount of slippage at contacts. Thus, the field of velocities and displacements becomes discontinuous, and on the whole element deformation (more accurately macrodeformation characterizing a change in angle β and sizes of elements in the directions Ox_1, Ox_2) consists of two parts: elastic, connected with elastic deformation of layers, and plastic, connected with slippage between them. Consequently, although shell material is ideally elastic, for analysis of its operation it is natural to use elastoplastic models, and those in which the essential features of the internal plastic deformation mechanism are considered: a discontinuous nature of the original displacement field, presence of slippage only in one direction, etc. These models were developed in [1, 2] where it was shown that the original discontinuous displacement field permits such smooth averaging that the strain tensor determined over the smoothed field characterizes macrodeformation, and in order to retain information about field discontinuities which is lost with averaging additional kinematic variables γ_{12} and γ_{21} are introduced having signifying dimensionless slippages. In order to determine the latter it is necessary to introduce internal rotation ω characterizing rotation of material microelements totally contained within a layer. In the case of an element (see Fig. 2) the set of definitive equations is written in the form

$$\varepsilon_{11}=\frac{\partial w_1}{\partial x_1}=\frac{1-\nu}{2\mu}\sigma_{11}-\frac{\nu}{2\mu}\sigma_{22},\ \varepsilon_{22}=\frac{\partial w_2}{\partial x_2}=\frac{1-\nu}{2\mu}\sigma_{22}-\frac{\nu}{2\mu}\sigma_{11},$$

$$\gamma_{12}=\frac{\partial w_2}{\partial x_1}-\omega=\frac{\sigma_{12}}{2\mu}+\Gamma(\sigma_{12}),\ \gamma_{21}=\frac{\partial w_1}{\partial x_2}+\omega=\frac{\sigma_{12}}{2\mu},$$

where μ is shear modulus; ν is Poisson's ratio; w_1 and w_2 are displacement vector components; Γ is a dimensionless value determining slippage.

An equation was written in [1] on a coordinate system whose lines coincided with slip lines. In the problem of deformation for a rolled shell this system may be described as follows. We take a circle of radius a and we draw tangent to it (Fig. 3). This circle is an envelope for a family of its tangents. Then from all of the points of the circle we draw spiral curves which at any point of the plane would be orthogonal to the straight line of the family of tangents corresponding to this point. Two reciprocally orthogonal families of lines are obtained, and it appears that distance h between neighboring turns of the same spiral is constant and equal to $2\pi a$.

Thus, in this case typical dimension h develops signifying the thickness of a layer of the shell. It should be noted that it is not necessary to consider h as a disappearing small value. This does not happen anywhere and it is not suggested. A changeover to averaging in an element of material signifies in fact that an elastic layer of the shell is a collection of disappearingly small elastic layers and total slippage between layers of thickness h is "spread" over the continuous element. It is also noted that in plotting distance h it is measured along the normal to the spiral, i.e., along the straight line of a family of tangents, and not along the radius as in an Archimedes spiral. This means that if a sheet of thickness h is taken for any material and it is wound into a roll, then the separating lines for layers of the shell obtained in fact coincide with the plotted spiral line.

Thus, coordinates of the grid have been plotted. We shall resolve the problem for annular region $R_0 \leqslant r \leqslant R$, $0 \leqslant \theta \leqslant 2\pi$ (see Fig. 3) and at internal boundary $r = R_0$ angle δ between a circle and the spiral is constant: $\delta = \arcsin(a/R_0)$. Selection of an annular region means that in a real shell (see Fig. 1) at the inner and outer boundaries with a changeover to circles part of the material is removed. Here a remarkable fact is noted: in a continuous arrangement the problem is axisymmetrical.

We select the following parameterization of the coordinate grid constructed: λ_2 is angle between the corresponding straight line of a family of tangents and axis Ox_1; λ_1 is length along a certain fixed line λ_2 = const. It appears to be sufficient to parameterize only section [AB] in order that the required spiral is determined from its family in terms of number λ_1. This selection of parameters means that in fixed line λ_2 = const it is necessary to fulfill congruence conditions: points $B(\lambda_1 + h, \lambda_2)$ and $C(\lambda_1, \lambda_2 + 2\pi)$ should coincide. Thus, transformation of polar coordinates (r, θ) to system (λ_1, λ_2) has the form

$$\lambda_1 = \sqrt{r^2 - \xi^2} - \xi(\theta - \arcsin(\xi/r)) - R_0 \cos\delta, \lambda_2 = \theta - \arcsin(\xi/r) \tag{2.1}$$

($\xi = R_0 \sin\delta$). Lamé parameters for this substitution are: $a_1 = 1$, $a_2 = \lambda_1 + \xi\lambda_2 + R_0 \cos\delta$

It is noted that the inclination angle $\varkappa$ of spiral λ_2 to the circle is not constant and at each point of the plane it is determined by the relationship $\tan \varkappa = \frac{\xi}{a_2} = \frac{\xi}{\sqrt{r^2 - \xi^2}}$. Radius of curvature ρ for spiral line λ_2 is also variable: $\rho = a_2 = \sqrt{r^2 - \xi^2}$. From this it follows that with movement of the shell along a spiral path bending moments of stress arise which however it is possible to ignore if it assumed that the average radius of the tube $(R + R_0)/2$ is much greater than the thickness of a shell layer h [3, 4].

System (2.1) was selected so that line λ_2 coincides with lines separating layers of the rolled shell. Then according to [1] the closed set of equations takes the form

$$\frac{\partial\sigma_{11}^0}{\partial\lambda_1} + \frac{\partial\sigma_{12}^0}{a_2\partial\lambda_2} + \frac{\sigma_{11}^0 - \sigma_{22}^0}{a_2} = 0, \quad \frac{\partial\sigma_{12}^0}{\partial\lambda_1} + \frac{\partial\sigma_{22}^0}{a_2\partial\lambda_2} + \frac{2\sigma_{12}^0}{a_2} = 0; \tag{2.2}$$

$$\frac{\partial w_1^0}{\partial\lambda_1} = \frac{1-\nu}{2\mu}\sigma_{11}^0 - \frac{\nu}{2\mu}\sigma_{22}^0, \quad \frac{\partial w_2^0}{a_2\partial\lambda_2} + \frac{w_1^0}{a_2} = \frac{1-\nu}{2\mu}\sigma_{22}^0 - \frac{\nu}{2\mu}\sigma_{11}^0; \tag{2.3}$$

$$\frac{\partial w_2^0}{\partial\lambda_1} + \frac{\partial w_1^0}{a_2\partial\lambda_2} - \frac{w_2^0}{a_2} = \frac{\sigma_{12}^0}{\mu} + \Gamma(\sigma_{12}^0, \lambda_1, \lambda_2), \tag{2.4}$$

where index 0 signifies projection on coordinates (λ_1, λ_2); (2.2) is an equilibrium equation on curvilinear coordinates; (2.3) characterizes inelastic changes in the dimensions of an elementary volume in directions λ_1 and λ_2; (2.4) describes shear strain of an elementary volume. Function Γ signifies the dimensionless value for slippage of layers and it may clearly depend on coordinates λ_1 and λ_2. This points to the possibility of taking account of inhomogeneous conditions at the contact (nonuniform lubrication). We limit ourselves below only to the direct problem when function Γ is previously prescribed from experimental data for the nature of the contact reaction of layers.

3. First we consider a rigorous statement of the problem when the shear modulus $\mu \to \infty$. In this case the shell is incompressible and inextendable, and the problem becomes kinematically determinate. System (2.3) is hyperbolic and it requires two boundary conditions for displacements. Let

$$w_1^0|_{\lambda_1=\chi_1} = 0, \quad w_2^0|_{\lambda_2=\chi_2} = U, \tag{3.1}$$

where U is a constant; $\lambda_1 = \chi_1(\lambda_2) = \xi(\sqrt{(R/\xi)^2 - 1} - \cot\delta - \lambda_2)$, $\lambda_2 = \chi_2(\lambda_1) = \sqrt{(R/\xi)^2 - 1} - \cot\delta - \lambda_1/\xi$ is external boundary of the region. This condition means that each point of the external boundary shifts along spiral λ_2 by distance U. Then from system (2.3) taking account of (3.1) we obtain

$$w_1^0 = 0, \quad w_2^0 = U. \tag{3.2}$$

From the condition $\gamma_{21}^0 = 0$ it follows that $\omega = U/a_2$. The results obtained point to the adequacy of the model constructed for real deformation of a rigid shell since this deformation

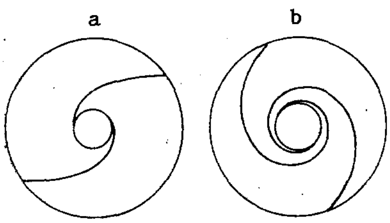

Fig. 4

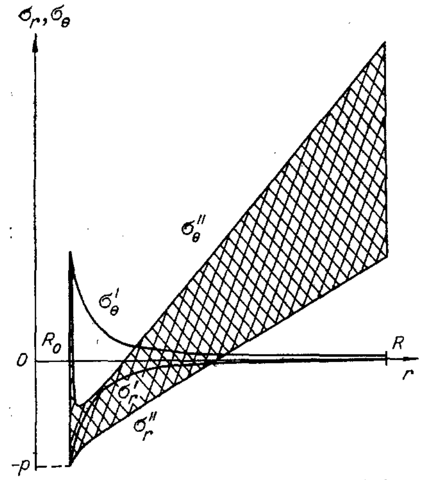

Fig. 5

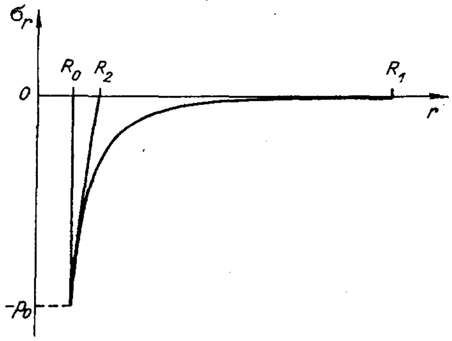

Fig. 6

may be understood as movement of elements of this shell along a prescribed spiral path, and the rotation angle ω for determination is the ratio of displacement U to radius of curvature of the path a_2. The results are illustrated in Figs. 4a, b (before the start of deformation the tube cross section is divided into parts by the diameter).

After displacements are found, by referring to known function Γ in (2.4) we find tangential stresses

$$\sigma^0_{12} = S(\lambda_1, \lambda_2). \tag{3.3}$$

By substituting (3.3) in (2.2) we obtain a solution for stresses in quadratures:

$$\begin{gathered}\sigma^0_{22} = \int \left(-2S(\lambda_1, \lambda_2) - a_2 \frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_1}\right) d\lambda_2 + f_1(\lambda_1), \\ \sigma^0_{11} = \frac{1}{a_2}\left(\int\left(-\frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_2}\right) d\lambda_1 + \iint\left(-2S(\lambda_1, \lambda_2) - a_2 \frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_1}\right) d\lambda_2 d\lambda_1 + \int f_1(\lambda_1)\, d\lambda_1 + f_2(\lambda_2)\right)\end{gathered} \tag{3.4}$$

[$f_i(\lambda_i)$ are arbitrary functions of integration]. For the boundaries we take the normal conditions of the form $\sigma_r|_{r=R_0} = -p$, $\sigma_{r\theta}|_{r=R_0} = 0$, and by reprojecting them on axis (λ_1, λ_2) from (3.4) we find that functions $f_i(\lambda_i)$ are determined from the relationships

$$\begin{gathered} f_1(\lambda_1) = -p - S\left(\lambda_1, -\frac{\lambda_1}{\xi}\right)\operatorname{ctg}\delta + \left[\int\left(2S(\lambda_1, \lambda_2) + a_2 \frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_1}\right) d\lambda_2\right]_{\lambda_2 = -\lambda_1/\xi}, \quad f_2(\lambda_2) = \\ = (-p - S(-\xi\lambda_2, \lambda_2)\operatorname{tg}\delta) R_0 \cos\delta + \left[\int \frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_2} d\lambda_1\right]_{\lambda_1 = -\xi\lambda_2} + \\ + \left[\iint\left(2S(\lambda_1, \lambda_2) + a_2 \frac{\partial S(\lambda_1, \lambda_2)}{\partial \lambda_1}\right) d\lambda_2 d\lambda_1\right]_{\lambda_1 = -\xi\lambda_2} - \left[\int f_1(\lambda_1)\, d\lambda_1\right]_{\lambda_1 = -\xi\lambda_2}. \end{gathered} \tag{3.5}$$

Thus, by prescribing a specific form for conditions at the contact [function $S(\lambda_1, \lambda_2)$] we have a solution for the original rigorous problem in the form of (3.2)-(3.5).

4. We consider the particular case of the original rigorous problem when the condition at the contact corresponds to the condition of constancy for tangential stresses

$$\sigma^0_{12} = -T, \quad T = \text{const}. \tag{4.1}$$

The system (3.4), (3.5), (4.1) gives the following stress distribution:

$$\begin{gathered}\sigma^0_{11} = -p - T\operatorname{ctg}\delta + T\left(\frac{a_2}{\xi} + \frac{\xi}{a_2}\right), \\ \sigma^0_{22} = -p - T\operatorname{ctg}\delta + 2T\frac{a_2}{\xi}, \quad \sigma^0_{12} = -T.\end{gathered} \tag{4.2}$$

In projection on polar coordinates expression (4.2) is transformed to the form

$$\sigma_r = -p + T\left(\sqrt{\left(\frac{r}{\xi}\right)^2 - 1} - \sqrt{\left(\frac{R_0}{\xi}\right)^2 - 1}\right),$$

$$\sigma_\theta = -p + T\left(\frac{\left(\frac{r}{\xi}\right)^2}{\sqrt{\left(\frac{r}{\xi}\right)^2 - 1}} + \sqrt{\left(\frac{r}{\xi}\right)^2 - 1} - \sqrt{\left(\frac{R_0}{\xi}\right)^2 - 1}\right), \sigma_{r\theta} = 0. \quad (4.3)$$

From (4.3) it follows that if it is assumed that T = 0, i.e., slippage of layers occurs freely ($\sigma_{12}^0 = 0$), then the solution coincides with the hydrostatic solution.

We consider the condition when all stresses occurring in a rolled tube do not exceed the elasticity limit for the material τ_s, for example in the sense of the criterion

$$\frac{1}{2}\sqrt{(\sigma_{11}^0 - \sigma_{22}^0)^2 + 4\sigma_{12}^{02}} < \tau_s, \quad (4.4)$$

which places a limitation on T in the form $T < \min\limits_{R_0 \leqslant r \leqslant R}\left(2\tau_s \frac{\xi}{r}\sqrt{1 - \left(\frac{\xi}{r}\right)^2}\right)$. This minimum depends essentially on angle δ (for the determination $\xi = R_0 \sin\delta$). It is clear that in order to improve the operation of the structure it is necessary to take such an angle δ with which the value of T is at a maximum. All of the aforementioned conditions lead to the situation that the supporting capacity of a rolled shell with a condition at the contact (4.1) will be at a maximum [in the sense of criterion (4.4)] with $\delta = \arcsin\left[R\big/\left(\sqrt{R^2 + R_0^2}\right)\right]$, and the limitation on T is transformed to the form $T < 2\tau_s R_0 R/(R_0^2 + R^2)$, which by substituting in distribution (4.3) and requiring that at the outer boundary of the region pressure equals zero (in view of the hyperbolic nature of this system this is not possible with all values of p), we obtain a limitation on internal pressure p.

Thus, the limiting pressure which the original shell may withstand without changing into a plastic condition (failure) with the condition at the contact (4.1)

$$p^* = 2\tau_s\left(1 - 2R_0^2/(R^2 + R_0^2)\right), \quad (4.5)$$

whereas in the Lamé solution [5] at the inner boundary of the region a plastic zone first develops with the load

$$p_0^* = \tau_s\left(1 - R_0^2/R^2\right). \quad (4.6)$$

By comparing expressions (4.5) and (4.6) we see that by allowing slippage in a thick-walled tube along spiral line λ_2 with condition at the contact (4.1) it is possible to obtain a gain in supporting capacity compared with a one-piece tube by almost a factor of two. It is clear that this becomes possible as a result of more uniform redistribution of applied load over the thickness of the structure, i.e., as a result of involving in the operation layers of the rolled shell distant from the inner edge (Fig. 5).

Now we prescribe some internal pressure p_0 and require that the material deforms only elastically [criterion (4.4)]. For a rolled shell a markedly smaller thickness is required $R_2 = R_0\sqrt{\frac{2 + p_0/\tau_s}{2 - p_0/\tau_s}}$, than for a one-piece tube $R_1 = R_0\frac{1}{\sqrt{1 - p_0/\tau_s}}$ (Fig. 6).

5. We consider the original problem taking account of elastic deformation of the shell layers themselves ($\mu < \infty$). As in the rigorous arrangement, we take the condition at the constant in the form of (4.1) (a condition for constancy of tangential stresses σ_{12}^0). It is evident that stress distribution will coincide with the distribution in the rigorous case since the problem is statically determinate and stresses are independent of kinematics. Displacements differ from the rigorous case by a contribution of elastic deformation of the layers themselves. From system (2.3) taking account of (4.2) after integration we have

$$\begin{aligned} w_1^0 &= \frac{1-2\nu}{2\mu}(-p - T\operatorname{ctg}\delta)\,a_2 + \frac{1-3\nu}{2\mu}T\frac{a_2^2}{2\xi} + \frac{1-\nu}{2\mu}T\xi\ln a_2 + \Phi_2(\lambda_2), \\ w_2^0 &= \frac{1-2\nu}{2\mu}Ta_2 + \frac{1-\nu}{2\mu}T\frac{a_2^3}{2\xi^2} - \frac{1-\nu}{2\mu}Ta_2\ln a_2 - \int\Phi_2(\lambda_2)\,d\lambda_2 + \Phi_1(\lambda_1) \end{aligned} \quad (5.1)$$

[$\Phi_i(\lambda_i)$ are arbitrary functions of integration]. If displacements of the outer shell boundary are prohibited $w_1^0|_{\lambda_1=\varkappa_1} = 0$, $w_2^0|_{\lambda_2=\varkappa_2} = 0$, then function $\Phi_i(\lambda_i)$ become constant and are determined from the relationships

$$\Phi_2(\lambda_2) = C_2 = \frac{1-2\nu}{2\mu}(P + T\operatorname{ctg}\delta)\,A - \frac{1-3\nu}{2\mu}T\frac{A^2}{2\xi} - \frac{1-\nu}{2\mu}T\xi\ln(A),$$

$$\Phi_1(\lambda_1) = C_1 = C_2 \frac{A}{\xi} - \frac{1-2\nu}{2\mu} TA - \frac{1-\nu}{2\mu} T \frac{A^3}{2\xi^2} + \frac{1-\nu}{2\mu} TA \ln(A) \quad (A = \sqrt{R^2 - \xi^2}). \tag{5.2}$$

Thus, expressions (4.2), (5.1), (5.2) give a complete solution for the elastoplastic arrangement of the original problem with conditions at the contact (4.1).

An approach has been considered for solving a class of elastic deformation problems for rolled shells based on using plastic models, and by plastic here we understand existence of slippage for layers of these shells. Analysis has been carried out for the stress–strain state of these structures. It appeared that as a result of the possibility of slippage of layers a rolled shell operates better than a one-piece thick-walled tube in the sense that it is possible to redistribute more uniformly the applied load through the thickness of the structure. In particular, if the condition at the contact is taken in the form of (4.1), then it is possible to obtain an advantage in supporting capacity by almost a factor of two compared with a one-piece tube. The model provided makes it possible to consider its generalization in a number of other models taking account for example of internal friction of the material, plastic deformation of the shell layers themselves, etc.

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ANTIPLANAR PLASTIC FLOW

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UDC 539.374

We will consider the equations describing nonsteady-state plastic flow of a Mises medium:

$$\begin{gathered} \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial p}{\partial x_i} + \frac{\partial s_{ij}}{\partial x_j}, \\ s_{ij}s_{ij} = 2k_s^2, \quad 2s_{ij} = \lambda\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \\ \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0, \end{gathered} \tag{1}$$

where u_1, u_2, u_3 are the components of the velocity vector, p is the hydrostatic pressure, λ is a nonnegative function, s_{ij} are the components of the stress tensor deviator, k_s is the yield point for pure shear, and repeating indices imply summation.

We will assume that the medium is located under conditions of antiplanar plastic flow, so that the solution of Eq. (1) will be sought in the form [1]

$$u_1 = 0,\ u_2 = 0,\ u_3 = w(x, y, t),\ p = 0. \tag{2}$$

Substituting Eq. (2) in Eq. (1) we obtain an equation describing antiplanar plastic flow:

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x} \frac{w_x}{\sqrt{w_x^2 + w_y^2}} + \frac{\partial}{\partial y} \frac{w_y}{\sqrt{w_x^2 + w_y^2}}$$

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